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Stability analysis of the pool-boiling crisis

A Steinchen and K Sefiane

Laboratoire de Modélisation en Mécanique et Thermodynamique, CP 322, Université Aix-Marseille 3, Faculté des Sciences St Jérôme, Boulevard Escadrille Normandie Niemen, 13397 Marseille Cédex 20, France

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Abstract. A stability analysis of the contact line at the bottom of vapour stems is undertaken in order to find out the dominant parameters responsible for the transition from nucleate boiling to film boiling. For strong constraints, the increase in the evaporation rate depletes the macrolayer and, as consequence, there is an enlargement of the dry areas. The second step of the boiling crisis study is to establish the relationship between the macrolayer depletion and the vapour column instability. The contact angle dynamics is very crucial to the occurrence of the crisis. When the macrolayer becomes unstable, the columns are cut off and consequently break down. We expect that before the macrolayer has been completely consumed the relative speed will have reached the critical value at which the Kelvin–Helmholtz instability of the vapour columns appears. It is clearly demonstrated that many kinds of instability participate in the boiling crisis.

1. Introduction

Heat and mass transfer in pool boiling remain most complicated phenomena, in spite of the considerable amount of work done on the subject. A theory describing the pool-boiling crisis was developed by Zuber (see [1]). This theory has gained wide acceptance among the scientific community. It has many shortcomings which were encountered in the experimental progress in the boiling field. Katto and Yokoya [2] introduced the existence of a liquid macrolayer at the base of the columns whose breaking down is related to the depletion of this macrolayer. New models have been developed to describe the steady state of the vapour stems in the macrolayer—see for instance the review papers by Lienard [3] and by Katto [4] and the recent paper by Lay and Dhir [5]. However, the occurrence of the instability which causes the enlargement of the dry areas remains a challenging problem. The stems feeding the vapour columns carrying heat through the liquid in the fully developed nucleate-boiling state of pool boiling are 'tied' to the heater by a liquid-evaporating meniscus. This meniscus is connected to the liquid macrolayer. The vapour recoil instability at the surface of the evaporating meniscus may lead to the expulsion of the macrolayer by a very fast expansion of the dry spot at the base of the stem. The contact angle of the liquid on the heater is a key parameter in the stability analysis of the evaporating meniscus.

2. The stability of an evaporating meniscus

For an evaporating wedge, the shape of the meniscus may be approximated by a cone tangential to the line of zero curvature in the vertical plane. In a 2D vertical cross section, the shape of the meniscus is a straight-line tangent to the inflexion point (see figure 1).

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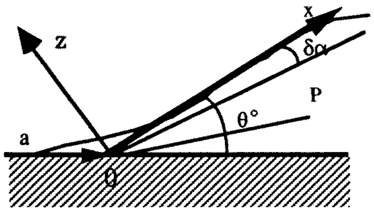

Figure 1. A 2D evaporating wedge.

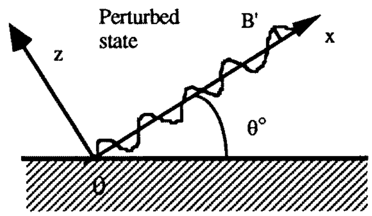

Figure 2. A 2D sine-perturbed evaporating wedge.

In the close vicinity of the evaporating surface, there exists a thermal layer of thickness $\delta_{th} = x \tan \delta\alpha \approx x\,\delta\alpha$, in which heat is transported only by conduction. For such a 2D system with a vapour front moving with a velocity $\mathrm{d}a/\mathrm{d}t$ and turning with an angular velocity $\mathrm{d}\theta/\mathrm{d}t$ it is easy to write the equations expressing the balances of the energy and momentum in the liquid in a coordinate system attached to the moving front and having the solid–liquid–vapour point as its origin. A stability analysis may then be performed by the standard method of linear perturbation around the steady state obtained when the flux of liquid from the macrolayer balances the flux of evaporation through the surface of the meniscus. In the perturbed state, the surface is slightly altered from its initial shape—by a sine mode (see figure 2).

A complete stability analysis of the thermal layer at a flat liquid–vapour interface with a thermal gradient normal to the surface has been performed by Palmer [6]. He has introduced a characteristic number (the Hickman number) for the vapour recoil instability. We have adapted his method to the geometry of the meniscus in the region where the surface is approximated by the tangential plane at the change of curvature (the inflexion point—see figure 1).

For stationary perturbations (exchange of stability), the dimensionless perturbation equations for momentum balances lead to the following expressions, where the pressure has been eliminated by twice taking the curl of the momentum equations:

$$(\tilde{D}^2 - \tilde{k}^2)(\tilde{D}^2 - \tilde{k}^2 - Re\,\tilde{D})\tilde{v}'_{z\ell} = 0 \qquad \text{in the liquid phase} \tag{1}$$

$$(\tilde{D}^2 - \tilde{k}^2)(\tilde{D}^2 - \tilde{k}^2 - Re\,N_\mu \tilde{D})\tilde{v}'_{zv} = 0 \qquad \text{in the vapour phase.} \tag{2}$$

The stationary-perturbation energy balance in the thermal layer reads

$$(\tilde{D}^2 - \tilde{k}^2 - Re\,Pr\,\tilde{D})\tilde{T}' = \tilde{v}'_{z\ell} \qquad 0 > \tilde{z} > -1 \tag{3}$$

$$(\tilde{D}^2 - \tilde{k}^2 - Re\,Pr\,\tilde{D})\tilde{T}' = 0 \qquad \text{outside the thermal layer } \left(-1 > \tilde{z} > -\frac{\tan\theta_0}{\delta\alpha}\right). \tag{4}$$

The solutions for the perturbation equations are obtained by the method of separation of variables assuming a single sine mode along the x-coordinate on the vapour–liquid surface. The solutions have to obey the following dimensionless boundary conditions.

(1) On the liquid–vapour surface (for $z = 0$):
(i) the relationship between the velocity perturbation in the vapour and the perturbation of the evaporation rate;
(ii) mass balance;
(iii) the continuity of the tangential velocity;
(iv) the normal-momentum balance;
(v) the tangential-momentum balance;
(vi) energy balance.

(2) On the heater surface (for $z = x \tan q_0$):
(a) all of the perturbations in velocity and temperature must vanish;
(b) the allowed dimensionless wave numbers must be odd multiples of $\pi\, \delta\alpha/2 \tan\theta_0$.

The following dimensionless numbers are introduced in the dimensionless differential equations and in the dimensionless boundary conditions:

the Hickman number: $$Hi = \left(\frac{\partial V_z}{\partial T}\right)\frac{V_z^* \beta (x\,\delta\alpha)^2 \mu_v}{D_{th}\sigma^*}\left[\frac{1}{\rho_v} - \frac{1}{\rho_\ell}\right]$$

the Marangoni number: $$Ma = -\left(\frac{\partial\sigma}{\partial T}\right)\frac{\beta(x\,\delta\alpha)^2}{\mu_\ell D_{th}}$$

the Reynolds number: $$Re = \frac{V_z^* \rho_\ell (x\,\delta\alpha)}{\mu_L}$$

the crispation number: $$Cr = \frac{\mu_\ell D_{th}}{\sigma^*(x\,\delta\alpha)}$$

the Prandtl number: $$Pr = \frac{\nu_\ell}{D_{th}}$$

the Bond number: $$Bo = \frac{(x\,\delta\alpha)^2 g(\rho_\ell - \rho_v)}{\sigma^*}$$

the Brinskman number: $$Br = \frac{V_z^* \nu_\ell^2}{\beta D_{th}(x\,\delta\alpha)^2}$$

the density ratio: $$N_\rho = \frac{\rho_\ell}{\rho_v}$$

the viscosity ratio: $$N_\mu = \frac{\mu_\ell}{\mu_v}$$

with β the unperturbed temperature gradient in the thermal boundary layer $\beta = \partial T/\partial z$, and $\partial\sigma/\partial T$ the temperature coefficient of the surface tension. Several of these depend on the position on the surface and on the contact angle θ_0 in the steady state. Indeed, among the above dimensionless numbers the Hickman, the Reynolds, and the Brinskman numbers are also functions of x and θ_0, through the steady evaporation rate V_z^*, as shown by Moosman and Homsy [7] (see figure 3).

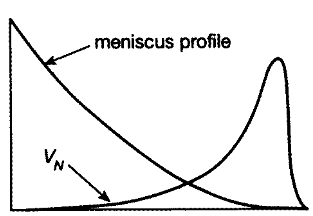

Figure 3. The evaporative normal flux V_N as a function of the dimensionless distance from the bulk of the meniscus (from Moosman and Homsy [7]).

This reveals that the meniscus instability will start in the neighbourhood of the contact line, where the evaporation flux is a maximum, and for a critical contact angle. The characteristic equation relates the local Hickman number to the dimensionless wavenumber ($k = k_x\,\delta\alpha$) and the other dimensionless groups in the marginal state (the neutral stability condition). A numerical resolution of the characteristic equation is now under way.

3. Conclusion—a relationship with the peak heat flux?

If, statistically, all the menisci at the bottom of the vapour stems (or of the bubbles) have reached a critical contact angle, the dry spots will grow and the vapour film will expand over the whole heater surface, consuming as it does this the whole macrolayer. Such cooperative instability cuts the vapour feed of the vapour columns, which will collapse, with the consequence that the transition to film boiling will take place, accompanied by a boiling crisis. The same scenario may occur under microgravity conditions—the only difference is that large bubbles remain on the heater and the expansion of the vapour at the bottom of the bubbles will favour the aggregation of large bubbles by the expulsion of the macrolayer. This could explain the peak heat flux observed also under low-gravity conditions.

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